

Physics data booklet

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Physics data booklet**

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Fundamental constants

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	9.81 m s^{-2}
Gravitational constant	G	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Avogadro's constant	N_A	$6.02 \times 10^{23} \text{ mol}^{-1}$
Gas constant	R	$8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
Boltzmann's constant	k_B	$1.38 \times 10^{-23} \text{ J K}^{-1}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Coulomb constant	k	$8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$
Permittivity of free space	ϵ_0	$8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ T m A}^{-1}$
Speed of light in vacuum	c	$3.00 \times 10^8 \text{ m s}^{-1}$
Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Elementary charge	e	$1.60 \times 10^{-19} \text{ C}$
Electron rest mass	m_e	$9.110 \times 10^{-31} \text{ kg} = 0.000549 \text{ u} = 0.511 \text{ MeV c}^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27} \text{ kg} = 1.007276 \text{ u} = 938 \text{ MeV c}^{-2}$
Neutron rest mass	m_n	$1.675 \times 10^{-27} \text{ kg} = 1.008665 \text{ u} = 940 \text{ MeV c}^{-2}$
Unified atomic mass unit	u	$1.661 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV c}^{-2}$
Solar constant	S	$1.36 \times 10^3 \text{ W m}^{-2}$
Fermi radius	R_0	$1.20 \times 10^{-15} \text{ m}$

Metric (SI) multipliers

Prefix	Abbreviation	Value
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deca	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Unit conversions

$$1 \text{ radian (rad)} \equiv \frac{180^\circ}{\pi}$$

$$\text{Temperature (K)} = \text{temperature (}^\circ\text{C)} + 273$$

$$1 \text{ light year (ly)} = 9.46 \times 10^{15} \text{ m}$$

$$1 \text{ parsec (pc)} = 3.26 \text{ ly}$$

$$1 \text{ astronomical unit (AU)} = 1.50 \times 10^{11} \text{ m}$$

$$1 \text{ kilowatt-hour (kWh)} = 3.60 \times 10^6 \text{ J}$$

$$hc = 1.99 \times 10^{-25} \text{ J m} = 1.24 \times 10^{-6} \text{ eV m}$$

Electrical circuit symbols

cell



battery



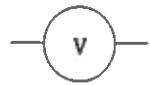
ac supply



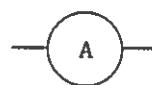
switch



voltmeter



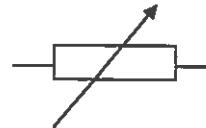
ammeter



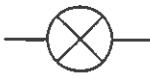
resistor



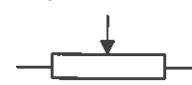
variable resistor



lamp



potentiometer



light-dependent resistor
(LDR)



thermistor



transformer



heating element



diode

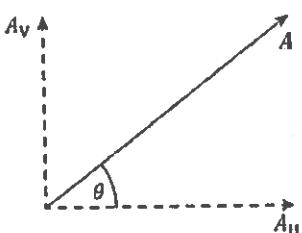


capacitor



Equations—Core

Note: All equations relate to the magnitude of the quantities only. Vector notation has not been used.

Sub-topic 1.2 – Uncertainties and errors	Sub-topic 1.3 – Vectors and scalars
<p>If: $y = a \pm b$ then: $\Delta y = \Delta a + \Delta b$</p> <p>If: $y = \frac{ab}{c}$ then: $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$</p> <p>If: $y = a^n$ then: $\frac{\Delta y}{y} = \left n \frac{\Delta a}{a} \right$</p>	 $A_H = A \cos \theta$ $A_V = A \sin \theta$

Sub-topic 2.1 – Motion	Sub-topic 2.2 – Forces
$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{(v+u)t}{2}$	$v = \text{final velocity}$ $u = \text{initial velocity}$ $s = \text{displacement}$ $F = ma$ $F_f \leq \mu_s R$ $F_f = \mu_d R$ $R = \text{normal force}$
Sub-topic 2.3 – Work, energy and power	Sub-topic 2.4 – Momentum and impulse
$W = Fs \cos \theta$ $E_K = \frac{1}{2}mv^2$ $E_P = \frac{1}{2}k\Delta x^2$ $\Delta E_P = mg\Delta h$ $\text{power} = Fv$ $\text{Efficiency} = \frac{\text{useful work out}}{\text{total work in}}$ $= \frac{\text{useful power out}}{\text{total power in}}$	$p = mv$ $F = \frac{\Delta p}{\Delta t}$ $E_K = \frac{p^2}{2m}$ $\text{Impulse} = F\Delta t = \Delta p$

Sub-topic 3.1 – Thermal concepts	Sub-topic 3.2 – Modelling a gas
$Q = mc\Delta T$ $Q = mL$ $C = \text{specific heat}$ $L = \text{specific latent heat}$	$p = \frac{F}{A} \leftarrow \text{equation for pressure}$ $n = \frac{N}{N_A}$ $n = \text{moles of gas}$ $N = \text{atoms of gas}$ $R = \text{ideal gas constant}$ $pV = nRT$ $N_A = \text{Avogadro's Number}$ $\bar{E}_K = \frac{3}{2} k_B T = \frac{3}{2} \frac{R}{N_A} T \leftarrow \text{avg. kinetic energy of a gas}$

Sub-topic 4.1 – Oscillations	Sub-topic 4.4 – Wave behaviour
$T = \frac{1}{f}$ $T = \text{period (s)}$ $f = \text{frequency (Hz)}$	$\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$ $s = \frac{\lambda D}{d}$
Sub-topic 4.2 – Travelling waves	Constructive interference: path difference = $n\lambda$ \leftarrow order of the maximum $(n=0 \text{ is central maximum})$
$c = f\lambda$ $c = \text{wave velocity}$	Destructive interference: path difference = $(n + \frac{1}{2})\lambda$ \leftarrow order of the minimum $(n=0 \text{ is first order minimum})$

Sub-topic 5.1 – Electric fields	Sub-topic 5.2 – Heating effect of electric currents
$I = \frac{\Delta q}{\Delta t}$ $I = \text{current (A)}$ $F = k \frac{q_1 q_2}{r^2}$ $q = \text{charge (C)}$ $k = \frac{1}{4\pi\epsilon_0}$ $k = \text{look it up}$ $V = \frac{W}{q}$ $E = \frac{V}{r}$ $r = \text{dist. between (m)}$ $E = \text{elec. field strength (N/C)}$ $I = nAvq$	Kirchhoff's circuit laws: $\sum V = 0 \text{ (loop)}$ $\sum I = 0 \text{ (junction)}$ $R = \frac{V}{I}$ $P = VI = I^2 R = \frac{V^2}{R}$ $R_{\text{total}} = R_1 + R_2 + \dots$ $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$ $\rho = \frac{RA}{L}$ $R - \text{resistance (\Omega)}$ $I - \text{current (A)}$ $V - \text{potential difference (V)}$ $P - \text{resistivity (\Omega m)}$ $A - \text{cross-sec. area (m}^2\text{)}$ $L - \text{wire length (m)}$

Sub-topic 5.3 – Electric cells	Sub-topic 5.4 – Magnetic effects of electric currents
$\epsilon = I(R + r)$	$F = qvB \sin \theta$ $F = BIL \sin \theta$

$\epsilon - \text{emf (aka potential diff)}$ $I - \text{current through battery}$ $R - \text{equivalent resistance of circuit}$ $r - \text{internal resistance of battery}$	$F = \text{magnetic force (N)}$ $q = \text{charge (C)}$ $v = \text{charge velocity (m/s)}$ $B = \text{mag. field strength (T)}$ $\theta = \text{angle in degrees}$ $I = \text{current (A)}$ $L = \text{length (m)}$
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Sub-topic 6.1 – Circular motion	Sub-topic 6.2 – Newton's law of gravitation
$v = \omega r$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ $F = \frac{mv^2}{r} = m\omega^2 r$ v - linear velocity (m/s) ω - angular velocity (rad/s) r - radius (m) T - period (s)	$F = G \frac{Mm}{r^2}$ $g = \frac{F}{m}$ $g = G \frac{M}{r^2}$

Sub-topic 7.1 – Discrete energy and radioactivity	Sub-topic 7.2 – Nuclear reactions
$E = hf$ $\lambda = \frac{hc}{E}$ E - energy (J) f - frequency (Hz) λ - wavelength (m) c - speed of light h - constant	$\Delta E = \Delta m c^2$ E - energy (J) m - mass (kg) c - speed of light (m/s)

Sub-topic 7.3 – The structure of matter

Charge	Quarks			Baryon number	Charge	Leptons		
$\frac{2}{3}q$	u	c	t	$\frac{1}{3}$	-1	e	μ	τ
$\frac{1}{3}q$	d	s	b	$\frac{1}{3}$	0	ν_e	ν_μ	ν_τ
All quarks have a strangeness number of 0 except the strange quark that has a strangeness number of -1								
Particles experiencing			Gravitational	Weak	Electromagnetic	Strong		
Particles mediating			All	Quarks, leptons	Charged	Quarks, gluons		
			Graviton	W^+, W^-, Z^0	γ	Gluons		

Sub-topic 8.1 – Energy sources	Sub-topic 8.2 – Thermal energy transfer
$\text{Power} = \frac{\text{energy}}{\text{time}}$ $\text{Power} = \frac{1}{2} A \rho v^3$ A - cross-sectional area (m^2) ρ - fluid density (kg/m^3) v - speed of fluid	$P = e\sigma AT^4$ $\lambda_{\max}(\text{metres}) = \frac{2.90 \times 10^{-3}}{T(\text{kelvin})}$ $I = \frac{\text{power}}{A}$ $\text{albedo} = \frac{\text{total scattered power}}{\text{total incident power}}$ P - power (W) e - emissivity σ - Stefan-Boltzmann constant A - area of emitting surface (m^2) T - temp. of emitting surface (K) I - intensity

Equations—AHL

Sub-topic 9.1 – Simple harmonic motion	Sub-topic 9.2 – Single-slit diffraction												
$\omega = \frac{2\pi}{T}$ $a = -\omega^2 x$ $x = x_0 \sin \omega t; x = x_0 \cos \omega t$ $v = \omega x_0 \cos \omega t; v = -\omega x_0 \sin \omega t$ $v = \pm \omega \sqrt{x_0^2 - x^2}$ $E_K = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$ $E_T = \frac{1}{2} m \omega^2 x_0^2$ l - length (m) g - acc. grav. m - mass (kg) k - sp. const. Rayleigh criterion for Circular Apertures	ω - angular frequency (rad/s) T - period (s) a - acceleration (m/s²) x - displacement from equilibrium (m) x_0 - max. amplitude t - time (s) v - inst. velocity v_0 - max velocity b - slit width (m) λ - wavelength (m) $\theta = \frac{\lambda}{b}$ - angle to first minimum (IN RADIANS!) Sub-topic 9.3 – Interference $n\lambda = d \sin \theta$ - equation for diffraction grating Constructive interference: $2dn = (m + \frac{1}{2})\lambda$ maxima locations Destructive interference: $2dn = m\lambda$ n - order of maxima ($n=0$ is central max) d - distance b/w lines on grating θ - angle (IN DEGREES!)												
Sub-topic 9.4 – Resolution $\theta = 1.22 \frac{\lambda}{R}$ $R = \frac{\lambda}{\Delta\lambda} = mN$ N - total # of slits illuminated	Sub-topic 9.5 – Doppler effect Moving source: $f' = f \left(\frac{v}{v \pm u_s} \right)$ Moving observer: $f' = f \left(\frac{v \pm u_o}{v} \right)$ $\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$												
Sub-topic 10.1 – Describing fields	Sub-topic 10.2 – Fields at work												
$W = q\Delta V_e$ $W = m\Delta V_g$ ΔV_e - charge in electrical pot. difference (V) m - mass (kg) ΔV_g - charge in gravitational pot. difference (J/kg)	<table border="1" style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td>$V_g = -\frac{GM}{r}$</td><td>$V_e = \frac{kq}{r}$</td></tr> <tr> <td>$g = -\frac{\Delta V_g}{\Delta r}$</td><td>$E = -\frac{\Delta V_e}{\Delta r}$</td></tr> <tr> <td>$E_p = mV_g = -\frac{GMm}{r}$</td><td>$E_p = qV_e = \frac{kq_1q_2}{r}$</td></tr> <tr> <td>$F_G = G \frac{m_1m_2}{r^2}$</td><td>$F_E = k \frac{q_1q_2}{r^2}$</td></tr> <tr> <td>$v_{esc} = \sqrt{\frac{2GM}{r}}$</td><td></td></tr> <tr> <td>$v_{orbit} = \sqrt{\frac{GM}{r}}$</td><td></td></tr> </tbody> </table>	$V_g = -\frac{GM}{r}$	$V_e = \frac{kq}{r}$	$g = -\frac{\Delta V_g}{\Delta r}$	$E = -\frac{\Delta V_e}{\Delta r}$	$E_p = mV_g = -\frac{GMm}{r}$	$E_p = qV_e = \frac{kq_1q_2}{r}$	$F_G = G \frac{m_1m_2}{r^2}$	$F_E = k \frac{q_1q_2}{r^2}$	$v_{esc} = \sqrt{\frac{2GM}{r}}$		$v_{orbit} = \sqrt{\frac{GM}{r}}$	
$V_g = -\frac{GM}{r}$	$V_e = \frac{kq}{r}$												
$g = -\frac{\Delta V_g}{\Delta r}$	$E = -\frac{\Delta V_e}{\Delta r}$												
$E_p = mV_g = -\frac{GMm}{r}$	$E_p = qV_e = \frac{kq_1q_2}{r}$												
$F_G = G \frac{m_1m_2}{r^2}$	$F_E = k \frac{q_1q_2}{r^2}$												
$v_{esc} = \sqrt{\frac{2GM}{r}}$													
$v_{orbit} = \sqrt{\frac{GM}{r}}$													

- ϕ - magnetic flux (Wb)
- B - magnetic field strength (T)
- A - cross sectional area (m^2)
- θ - angle between NORMAL to surface + B-field ($^\circ$)

Sub-topic 11.1 – Electromagnetic induction	Sub-topic 11.3 – Capacitance
$\phi = BA \cos \theta$ $E = -N \frac{\Delta \phi}{\Delta t}$ $E = Bvl$ $E = BvlN$	$C = \frac{q}{V}$ $C_{\text{parallel}} = C_1 + C_2 + \dots$ $\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ $C = \epsilon \frac{A}{d}$ $E = \frac{1}{2} CV^2$ $\tau = RC$ $q = q_0 e^{-\frac{t}{\tau}}$ $I = I_0 e^{-\frac{t}{\tau}}$ $V = V_0 e^{-\frac{t}{\tau}}$
Sub-topic 11.2 – Power generation and transmission	ϵ - permittivity of material between plates $(\text{if air or vacuum use } \epsilon_0)$ E - electric field strength τ - time constant of RC circuit R - resistance (Ω) t - time (s)
$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ rms - root mean square $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ \bar{P} - average power $R = \frac{V_0}{I_0} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$ $P_{\text{max}} = I_0 V_0$ $\bar{P} = \frac{1}{2} I_0 V_0$ $\frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$	<p>equations for charge/current/pot. diff. left in an RC circuit after a set time has elapsed.</p>

Sub-topic 12.1 – The interaction of matter with radiation	Sub-topic 12.2 – Nuclear physics
$E = hf$ $E_{\text{max}} = hf - \Phi$ $E = -\frac{13.6}{n^2} \text{ eV}$ $mvr = \frac{nh}{2\pi}$ $P(r) = \psi ^2 \Delta V$ $\Delta x \Delta p \geq \frac{\hbar}{4\pi}$ $\Delta E \Delta t \geq \frac{\hbar}{4\pi}$	$R = R_0 A^{1/3}$ $N = N_0 e^{-\lambda t}$ $A = \lambda N_0 e^{-\lambda t}$ $\sin \theta \approx \frac{\lambda}{D}$ θ - angle ($^\circ$) λ - wavelength D - nuclear diameter

$P(r)$ - probability of particle location

ψ - wave function

ΔV - change in potential

Δx - uncertainty in position

Δp - uncertainty in momentum

ΔE - uncertainty in energy

Δt - uncertainty in time

\hbar - constant

N - current population/activity

N_0 - original population/activity

$$\lambda - \text{decay constant} \rightarrow T_{1/2} = \frac{\ln 2}{\lambda}$$

t - time (s)

↑
Memorize this!!!

Equations—Options

Sub-topic A.1 – The beginnings of relativity	Sub-topic A.2 – Lorentz transformations
$x' = x - vt$ $u' = u - v$	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
Sub-topic A.3 – Spacetime diagrams	$x' = \gamma(x - vt)$; $\Delta x' = \gamma(\Delta x - v\Delta t)$ $t' = \gamma(t - \frac{vx}{c^2})$; $\Delta t' = \gamma(\Delta t - \frac{v\Delta x}{c^2})$ $u' = \frac{u - v}{1 - \frac{uv}{c^2}}$ $\Delta t = \gamma\Delta t_0$ $L = \frac{L_0}{\gamma}$ $(ct')^2 - (x')^2 = (ct)^2 - (x)^2$
Sub-topic A.4 – Relativistic mechanics (HL only)	Sub-topic A.5 – General relativity (HL only)
$E = \gamma m_0 c^2$ $E_0 = m_0 c^2$ $E_K = (\gamma - 1)m_0 c^2$ $p = \gamma m_0 v$ $E^2 = p^2 c^2 + m_0^2 c^4$ $qV = \Delta E_K$	$\frac{\Delta f}{f} = \frac{g\Delta h}{c^2}$ $R_s = \frac{2GM}{c^2}$ $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$

Sub-topic B.1 – Rigid bodies and rotational dynamics	Sub-topic B.2 – Thermodynamics
$\Gamma = Fr \sin \theta$ $I = \sum mr^2$ $\Gamma = I\alpha$ $\omega = 2\pi f$ $\omega_f = \omega_i + at$ $\omega_f^2 = \omega_i^2 + 2a\theta$ $\theta = \omega_i t + \frac{1}{2}at^2$ $L = I\omega$ $E_{K_{rot}} = \frac{1}{2}I\omega^2$	Γ - torque (Nm) F - force (N) r - length of moment arm (m) I - moment of inertia (kg m^2) a - angular acc. (rad/s^2) ω - angular velocity (rad/s) L - angular momentum ($\text{kg m}^2/\text{s}$) E - energy (J)
	$Q = \Delta U + W$ $U = \frac{3}{2}nRT$ $\Delta S = \frac{\Delta Q}{T}$ $pV^{\frac{5}{3}} = \text{constant (for monatomic gases)}$ $W = p\Delta V$ $\eta = \frac{\text{useful work done}}{\text{energy input}}$ $\eta_{\text{Carnot}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$
	Q - thermal energy in/out (J) ΔU - internal energy (J) W - work done on/by gas (J) η - efficiency
Sub-topic B.3 – Fluids and fluid dynamics (HL only)	Sub-topic B.4 – Forced vibrations and resonance (HL only)
$B = \rho_f V_f g$ $P = P_0 + \rho_f g d$ $Av = \text{constant}$ $\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$ $F_D = 6\pi\eta r v$ $R = \frac{\nu r \rho}{\eta}$	B - buoyant force (N) ρ - density (kg/m^3) V_f - volume displaced (m^3) P - pressure F_D - drag force (N) η - viscosity r - radius v - velocity
	$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$ $Q = 2\pi \times \text{resonant frequency} \times \frac{\text{energy stored}}{\text{power loss}}$ Q - Q - Factor
Sub-topic C.1 – Introduction to imaging	Sub-topic C.2 – Imaging instrumentation
$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $P = \frac{1}{f}$ $m = \frac{h_l}{h_o} = -\frac{v}{u}$ $M = \frac{\theta_l}{\theta_o}$ $M_{\text{near point}} = \frac{D}{f} + 1; M_{\text{infinity}} = \frac{D}{f}$	$M = \frac{f_o}{f_e}$
Sub-topic C.3 – Fibre optics	
	$n = \frac{1}{\sin c}$ $\text{attenuation} = 10 \log \frac{l}{l_0}$
Sub-topic C.4 – Medical imaging (HL only)	
	$L_l = 10 \log \frac{l_1}{l_0}$ $I = I_0 e^{-\mu x}$ $\mu x_{\frac{1}{2}} = \ln 2$ $Z = pc$

R - Reynolds' number

Sub-topic D.1 – Stellar quantities	Sub-topic D.2 – Stellar characteristics and stellar evolution
$d \text{ (parsec)} = \frac{1}{p \text{ (arc-second)}}$ $L = \sigma A T^4$ $b = \frac{L}{4\pi d^2}$	$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m K}$ $L \propto M^{3.5}$
Sub-topic D.3 – Cosmology	Sub-topic D.5 – Further cosmology (HL only)
$z = \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$ $z = \frac{R}{R_0} - 1$ $v = H_0 d$ $T \approx \frac{1}{H_0}$	$v = \sqrt{\frac{4\pi G \rho}{3}} r$ $\rho_c = \frac{3H^2}{8\pi G}$

