Investigating the Deflection of Beams Under a Load

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CLEAR PERSONAL ENGAGEMENT BOTH IN CONNECTION TO PERSONAL LIFE AND THE CARE FOR DETAIL GIVEN. STUDENT EVEN WENT TO NEARBY UNIVERSITY TO GAIN ACCESS TO A DIAL GAUGE.

1. Introduction

Being able to predict the deflection of a beam when it is subjected to a load is a very valuable tool when deciding on building materials. Although beam deflection does not generally create a safety risk in itself, excessive deflection might impair structures in other ways. Namely, it might cause cracks in building walls or cause floors to sag when they are walked on. When building machines it is also very important that beams deflect just the right amount as to allow parts such as gears to make proper contact with one another (Philpot, 2008). In addition, excessive deflection can cause unwanted vibrations in machine parts. I recently learned about the importance of deflection from my mother, a civil engineer, who taught me about it after I expressed a desire to explore different aspects of engineering (a field that I plan on studying in university). After researching and learning about how deflection is calculated, I began to wonder how actual beams of certain materials would perform under a load as compared to the theoretical calculations.

2. Background

In order to calculate the deflection of a beam, three things must first be known about the beam: the elastic modulus of its material (E), the beams length (L), and its cross-sectional dimensions. It must also be known that the deflection is also a function of the load applied. For a simply-supported beam that is loaded by a single load (P) at mid-span, the deflection can be calculated from Equation 1 below (Philpot, 2008):

$$v = -\frac{PL^3}{48EI} \qquad [1]$$

In this equation, v represents the deflection of the beam at its lowest point and is given in units of distance, P represents the load placed on the beam and is given in units of force, L represents length of the beam and is given in units of distance, E represents the elastic modulus of the material and is given in units of pressure, and I represents the moment of inertia of the beam and is given in units of distance to the fourth power (Philpot, 2008).

The elastic modulus is a material property and is a measure of the material stiffness or resistance to axial loading. It can be determined by conducting an axial load test, or tension test. When a tensile load is applied, the material will get longer. However, the length of the beam, L, and its cross-sectional area, A, affects the relationship between the load and the resulting elongation. Consequently, in order to determine E, the force and

RELEVANT AND APPROPRIATE BACKGROUND WHICH AIDS READER IN UNDERSTANDING THE CONTEXT OF THIS IA.

elongation must be normalized. This is achieved by dividing the load, P, by the cross-sectional area of the beam, A, and the change in length, ΔL, by the original length, L. Here P/A is known as the stress, σ, and $\Delta L/L$ is known as the strain, ε. The elastic modulus is then the initial slope of the stress-strain plot (C. Human, personal communication, 2016). Because the elastic modulus is an important material property, its value for many materials is often readily available in textbooks or online. For example, the elastic modulus of aluminum is, depending on where one looks, between 69,000 and 70,000N•mm^{-2} (Philpot, 2008), and the elastic modulus of pinewood ranges from 9,000 to 16,000N•mm^{-2} ("Modulus of Elasticity or Young's Modulus")("Compressive Strength of Spruce-Pine-Fir Softwood").

The moment of inertia of an beam is a measure of the distribution of the material away from the horizontal axis of symmetry of its cross sectional area. For example, Figure 1 shows the cross sections of two identical beams oriented differently. Because the beam on the left has more material further away from the dashed line denoting the horizontal axis of symmetry, it will have a larger moment of inertia than the beam on the right. Equation 2 defines moment of inertia is as follows (C. Human, personal communication, 2015):

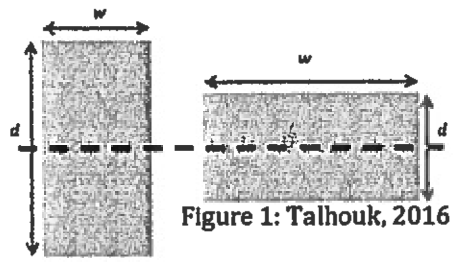

Figure 1: Talhouk, 2016

$$I = \frac{1}{12}wd^3 \qquad [2]$$

In this equation, w denotes the width of the beam given in units of distance, and d denotes the depth of the beam, also given in distance.

In summary, referring back to Equation 1, as the load on the beam and the length of the beam increase, so does the expected deflection. Conversely, as the elastic modulus of the material and the moment of inertia of the beam increase, the expected deflection decreases.

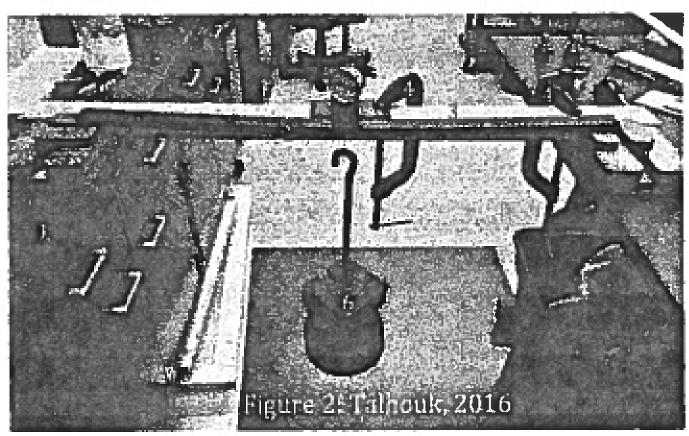

Figure 2: Talhouk, 2016

3. Materials and Methods

In order to test how beams of certain materials react under load, a small apparatus was set up (Figure 2). The set-up consisted of two tabletops of the same height and 864 ±

2mm apart (#1 in Figure 2), a stand and dial gauge to measure deflection (#2), a wood beam to support the dial gauge stand (#3), clamps to hold in place the aluminum and dial gauge (#4), the beam to be tested (#5), and a hanger with weights (#6). The dial gauge used is rated to be accurate to 0.0254cm.

The two beams tested in this investigation were made of pinewood and hollow aluminum respectively. The dimensions of the pine are 5.44 ± 0.1mm by 34.3 ± 0.2mm. The outer dimensions of the aluminum are 38.1 ± 0.1mm by 15.8 ± 0.1mm, and the beam has a wall thickness of 1.00 ± 0.05mm. The dimensions of the beams were taken using calipers (accurate to ± 0.0254mm) at six points along the beam and an average was taken to arrive at one value for each dimension. Because the measurement uncertainty is relatively small compared to the variability found in the width of the beams, the uncertainty in the dimension is based on that variability.

Once measured, the beams were between the tabletops and a loop of wire was placed in the center of the beam, creating a point load. Masses were then hung from the loop of wire and deflection was measured using the dial gauge as shown in Figure 2. The mass differed between each beam in order to observe measureable deflection without causing the beam to fail. The deflection of the both beams was measured on both axes in order to determine the effect of moment of inertia.

4. Data

After data collection, the masses and deflections for each beam were collected into the table below (Table 1). The mass of the objects hung from the beams was converted into force through the equation $F = ma$ where a was the gravitational constant of 9.81ms^{-2}. This was done in order to compare the experimental results with the theoretically correct results to be derived from Equation 1, which requires the load be given in units of force. Deflection curves for each beam and axis were then developed (Figures 3-6) and best-fit lines inserted to model the curve. Uncertainty slopes were not included because (1) on some graphs, uncertainty was so small that error bars did not protrude past the data points, and (2) maximum and minimum slopes would have intercepts other than zero, which is impossible given that a zero load value must mean zero deflection.

EXCELLENT RAW AND PROCESSED DATA.

Table 1		
Wood, Broad Side		
Total Mass	Total Force	Total Deflection
kg	N	mm
$\Delta m = \pm 0.0001$kg	$\Delta P = \pm 0.000981$N	$\Delta v = \pm 0.127$mm
0.207	2.03067	3.937
0.4125	4.046625	8.001
0.9023	8.851563	6.7132
1.1078	10.867518	19.7866
Wood, Narrow Side		
Total Mass	Total Force	Total Deflection
kg	N	mm
$\Delta m = \pm 0.0001$kg	$\Delta P = \pm 0.000981$N	$\Delta v = \pm 0.127$mm
0.9023	8.851563	0.635
1.9073	18.710613	1.2954
2.9049	28.497069	1.8288
3.9039	38.297259	2.2098
4.9061	48.128841	2.6162
Aluminum, Broad Side		
Total Mass	Total Force	Total Deflection
kg	N	mm
$\Delta m = \pm 0.0001$kg	$\Delta P = \pm 0.000981$N	$\Delta v = \pm 0.127$mm
0.9023	8.851563	0.4318
11.1183	109.070523	5.2197
15.1265	148.390965	6.8834
19.1191	187.558371	8.636
23.1145	226.753245	10.414
Aluminum, Narrow Side		
Total Mass	Total Force	Total Deflection
kg	N	mm
$\Delta m = \pm 0.0001$kg	$\Delta P = \pm 0.000981$N	$\Delta v = \pm 0.127$mm
0.9023	8.851563	0.1143
11.1183	109.070523	1.3462
17.1126	167.874606	2.0828
23.0999	226.610019	2.794

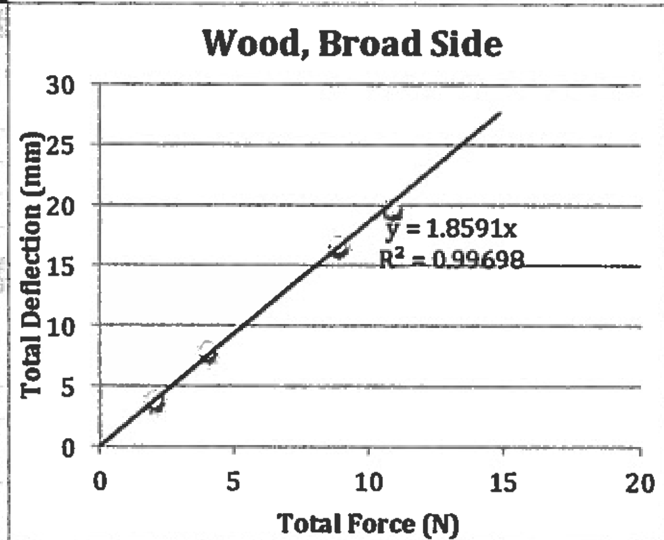

Figure 3: R. Talhouk

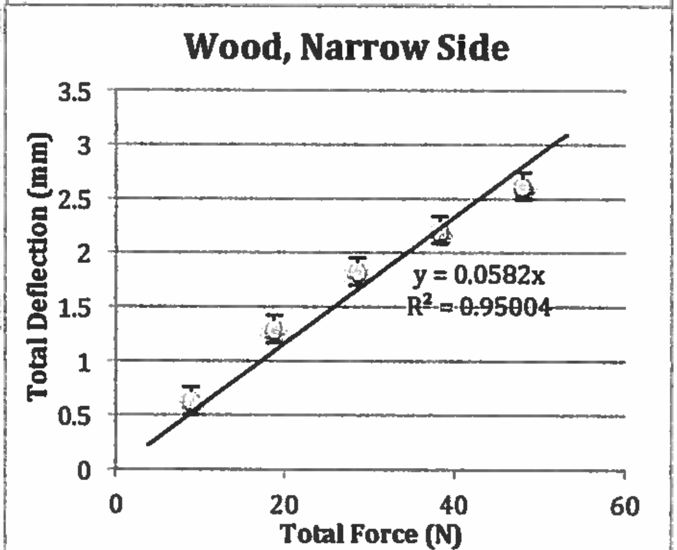

Figure 4: R. Talhouk

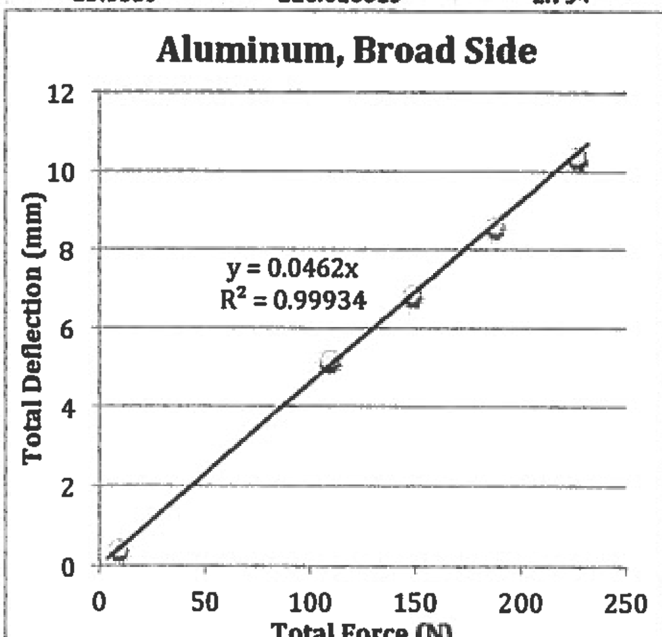

Figure 5: R. Talhouk

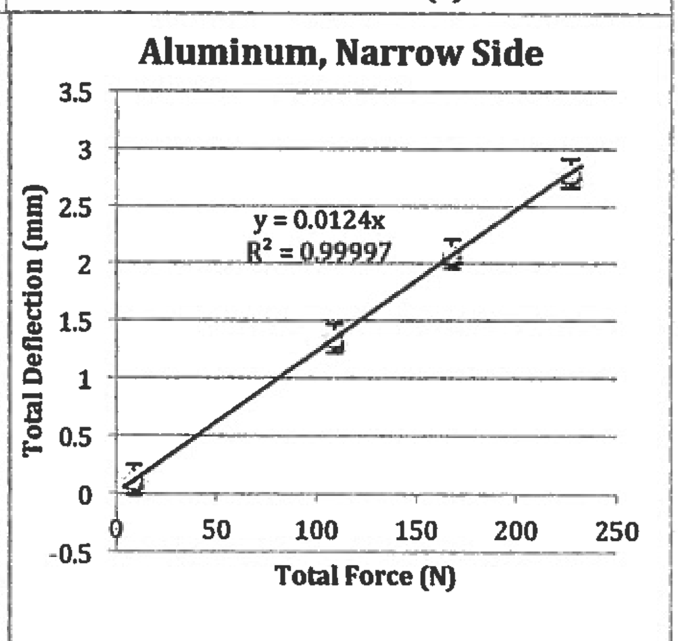

Figure 6: R. Talhouk

5. Calculations

With the data tabulated and graphed, the theoretically correct deflection of each beam on each axis can easily be calculated using the equations for moment of inertia and deflection, seen below as Equations 2 and 1 respectively:

[2] $I = \frac{1}{12}wd^3$ $v = -\frac{PL^3}{48EI}$ [1]

Table 2	
Beam, Axis	**Moment of Inertia**
Wood, Broad Side	$457.5 \pm 28.1 mm^4$
Wood, Narrow Side	$17975 \pm 657 mm^4$
Aluminum, Broad Side	$4595 \pm 212 mm^4$
Aluminum, Narrow Side	$18717 \pm 703 mm^4$

Because a known moment of inertia is needed in order to calculate deflection, the moment of inertia was calculated using Equation 2 and shown in Table 2 to the right. Because the aluminum is a hollow beam, the moment of inertia is calculated by subtracting the moment of inertia of the interior dimensions from that of the exterior dimensions (Philpot, 2008).

Now that I has been calculated, it can be plugged into Equation 1 in order to figure out the theoretically correct deflection for each beam and orientation using the load and length measured, and the elastic moduli found online. The results of these calculations were added to the deflection curves (Figures 7-10) of the experimental data for comparison. The uncertainty for both the moment of inertia and deflection calculations was derived using the method set forth by the International Baccalaureate: if $y = \frac{ab}{c}$, then $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$. Therefore, because $I = \frac{1}{12}wd^3$, then $\frac{\Delta I}{I} = \frac{\Delta w}{w} + 3\frac{\Delta d}{d}$; also, because $v = -\frac{PL^3}{48EI}$, then $\frac{\Delta v}{v} = \frac{\Delta P}{P} + 3\frac{\Delta L}{L} + \frac{\Delta E}{E} + \frac{\Delta I}{I}$.

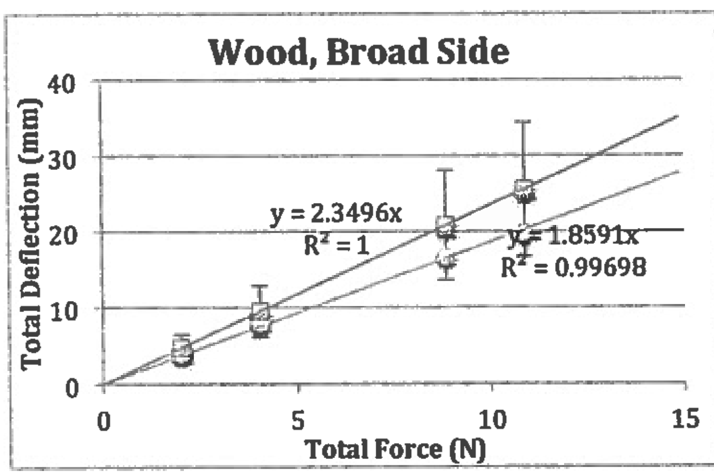

Figure 7: R. Talhouk, 2016

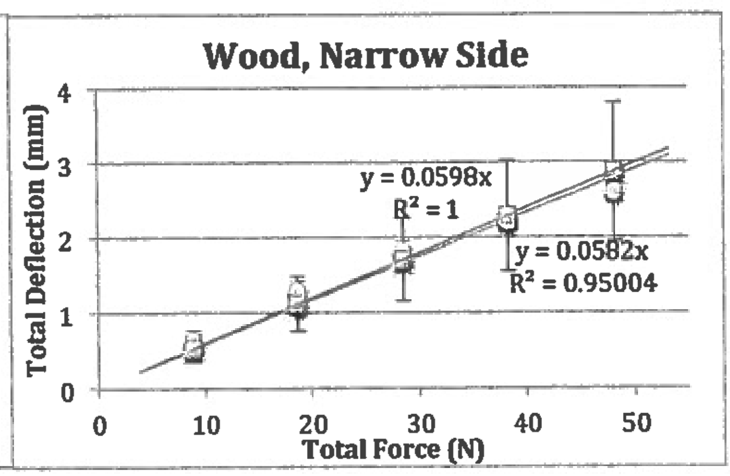

Figure 8: R. Talhouk, 2016

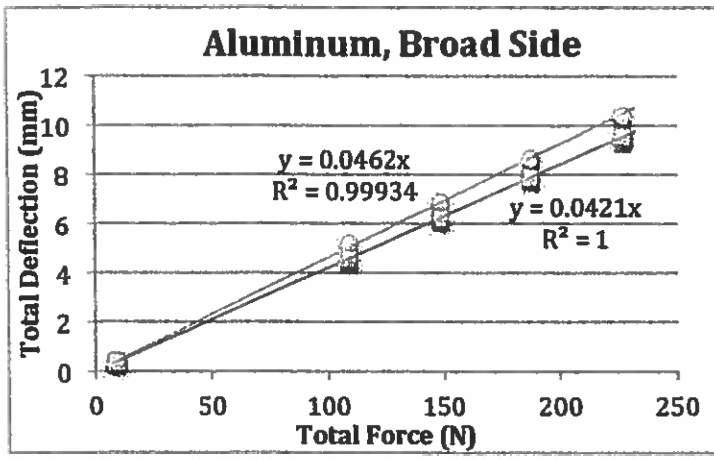

Figure 9: R. Talhouk, 2016

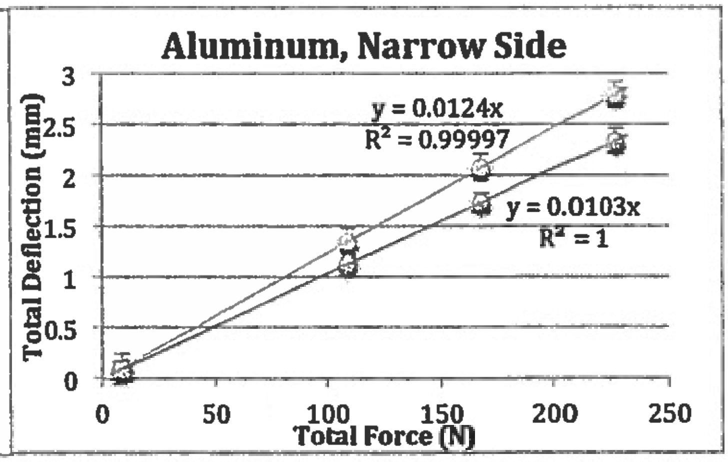

Figure 10: R. Talhouk, 2016

6. Analysis

[illegible] data comparing the experimental to theoretical data shows that one observation can be easily made. Namely, the data shows that the experimental deflection of each beam could not be modeled perfectly. There are several reasons why this could be the ca[illegible]ll have to do with the uncertainty with which the theoretical values were calculated, such as in the elastic modulus and moment of inertia.

The elastic modulus of a material is quite specific, even varying from beam to beam if only slightly. Moreover, the elastic moduli of the beams tested in this investigation were not known as they were purchased commercially. Therefore, the elastic moduli, as mentioned in Section 2, were found online and in textbooks. It was found that each online or book source provided a slightly different elastic modulus. To take into account all these sources, the elastic moduli for this investigation were found by taking the midpoint between the highest and lowest values found (ranges for each of the materials were mentioned in Section 2) and the uncertainly was the value between the midpoint and each extreme. For example, the elastic modulus of the pinewood was calculated to be 12500 $\pm$ 3500N•mm^{-2}, and the aluminum 69500 $\pm$ 500N•mm^{-2}. This was cause for much more uncertainty in the wood's deflection because, as wood's strength can vary from tree to tree, there is naturally a higher range for its elastic modulus. However, due to this unpredictability in uncertainty, experimental deflections were able to fall within the error bars of the calculated deflections (Figures 7-8).

RESULTS ARE DESCRIBED AND JUSTIFIED, THEN COMPARED TO ACCEPTED SCIENTIFIC CONTEXT

Moment of inertia also was very sensitive to change when doing these calculations given the fact that, based on the equation $I = \frac{1}{12}wd^3$, the depth of the beam is cubed. Therefore, a small mistake when measuring the dimensions of a beam could lead to distorted calculations. Moment of inertia calculations were likely affected in this investigation by the fact that the beams were not the same dimensions all the way through their lengths. Furthermore, because the aluminum beam is hollow, the wall thickness could only be measured on the outside, while in reality the thickness could vary as it nears the center of the beam.

It was also observed that while the data points did not model the expected deflections exactly, they all were off by roughly the same factor. Namely, throughout all of the data points collected in this investigation, the expected values and the experimental values were off by an average factor of 1.16, with a standard deviation of 0.07.

7. **Limitations**

When conducting experiments, it is always difficult to garner data that is perfectly reliable; there are always some limitations that could affect the data. This investigation is no exception. Throughout data collection and analysis, a few problems were come across that could have potentially skewed the data one way or another.

Firstly, when measuring the dimensions of the aluminum beam, the wall thickness could only be measured at the ends of the beam, making it impossible to know whether the wall thickness varied throughout the length of the beam. This is significant because even a slight change in wall thickness could materially impact the moment of inertia and subsequently the measured deflection.

Secondly, when loading the wood on its narrow side, there was no way to clamp it down on its edges. Therefore, the beam had to be held still by hand, opening the door for some error in the deflection reading due to small movements in the beam caused by the human hand.

Finally, and perhaps the most significant limitation, was the lack of resources to do a stress test on the beams. This is a significant limitation because, as mentioned, it is a stress test that allows for the elastic modulus of a beam to be calculated. Without a stress test, the

STRENGTHS AND WEAKNESSES ARE DESCRIBED AND A CLEAR UNDERSTANDING OF METHODOLOGICAL ISSUES INVOLVED IS DISPLAYED elastic modulus had to be found online and was the source of much of the uncertainty in the theoretical deflection calculations.

8. Conclusions

After analyzing all the data collected, a conclusion can be drawn about comparing expected and experimental beam deflection can be drawn. Namely, while the deflection of one particular beam cannot be exactly predicted with ease, it can be approximated to a relatively consistent degree of accuracy and modeled using a linear trend line. Also, while it cannot be concluded with certainty, based on the data gathered, it can be hypothesized that the difference between expected and experimental deflection has to do with error in the elastic module or moment of inertia (due to imperfections) of the beam being studied.

9. Areas of Further Study

When researching elastic modulus and searching for the specific modulus of each beam material used in this investigation, the principles of elastic modulus remained constant from source to source, but the values for different material varied considerably. I then got curious as to how much elastic modulus changes between beams of the same material and what factors or properties of a beam would cause its elastic modulus to change. If I were to continue this investigation, I would perform stress tests on beams of the same material in order to see if elastic modulus differs from beam to beam. This will also hopefully shed light on the cause of all the error within the elastic moduli found online and allow the theoretical deflection calculations of this investigation to more accurately reflect the beams used rather than an unspecific range.

REALISTIC EXTENSIONS ARE DISCUSSED

10. Works Cited

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